The evolution of core and surface magnetic field in isolated neutron stars

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ABSTRACT

We apply the model of flux expulsion from the superfluid and superconductive core of a neutron star, developed by Konenkov & Geppert (2000), both to neutron star models based on different equations of state and to different initial magnetic field structures. When initially the core and the surface magnetic field are of the same order of magnitude, the rate of flux expulsion from the core is almost independent of the equation of state, and the evolution of the surface field decouples from the core field evolution with increasing stiffness. When the surface field is initially much stronger than the core field, the magnetic and rotational evolution resembles to those of a neutron star with a purely crustal field configuration; the only difference is the occurence of a residual field. In case of an initially submerged field significant differences from the standard evolution occur only during the early period of neutron star's life, until the field has been rediffused to the surface. The reminder of the episode of submergence is a correlation of the residual field strength with the submergence depth of the initial field. We discuss the effect of the rediffusion of the magnetic field on to the difference between the real and the active age of young pulsars and on their braking indices. Finally, we estimate the shear stresses built up by the moving fluxoids at the crust–core interface and show that preferentially in neutron stars with a soft equation of state these stresses may cause crust cracking.

Key words: magnetic fields - stars: neutron - pulsars: general - stars: evolution

1 INTRODUCTION

Since the process of the generation of neutron star (NS) magnetic fields (MFs) and, hence, their initial structure, strength and localization is still under discussion, we here intend to investigate the evolution of NS MFs which penetrate the entire star. Though there are arguments that the NS MF may be confined to the crustal layer, where it has been generated by thermoelectric effects during the early hot period of the NS's life when the temperature gradients in the crust are immense (Blandford, Applegate & Hernquist 1983; Urpin, Levshakov & Yakovlev 1986; Wiebicke & Geppert 1996), one has also to consider the possibility that the NS MF permeates both the crust and the core of the star. This could be the field structure from the very beginning of the NS's life, either according to the simple model of flux conservation during the collapse or due to a very efficient dynamo action in the core of the proto–NS as described by Thompson & Duncan (1993).

In a recent paper (Konenkov & Geppert 2000, hereafter KG00) we considered the flux expulsion from the superfluid core of a NS. Since the transition from the normal to the superfluid state of the matter in the cores of NSs may occur rather early after the NS's birth (Page 1998), we assumed that the protons in the core form a superconductor of type II (Baym, Pethick & Pines 1969) and the magnetic flux is concentrated in an array of proton flux tubes (fluxoids). There are several forces acting on to the fluxoids (for details see Sec. 2), driving them outward into the normal conductive crust of a NS, where the magnetic flux may decay ohmically. Considering an initial field structure where the MF at the surface and in the core has the same strength, we showed that the field evolution in the crust leads to a deceleration of the flux expulsion which is the more pronounced the stronger the initial surface MF is. It turned out, that the main force, which is responsible for the flux expulsion from the core, is the buoyancy force.

We have also confirmed the result of Ding, Cheng & Chau 1993 (hereafter DCC), that not the entire magnetic flux is expelled from the core, but a certain part of it remains there for eternity. This results in a nonvanishing residual field B_{res}, which can be in the range of $10^7 - 10^{10}$ G, depending on the model parameters.

It is well known that the equation of state (EOS), which describes the state of matter in the core region of the NS, has a crucial influence on the crustal MF decay in isolated NSs (Urpin & Konenkov 1997; Page, Geppert & Zannias 2000). The EOS determines the compactness of the NS and, hence, the scale length of the MF in the crust, as well as the cooling history of the NS. While a softer EOS causes an smaller scale which, in turn, leads to a more rapid field decay, a softening of the EOS decelerates also the cooling. In a warmer crust the MF will be dissipated faster too. Since the EOS influences also - via the moment of inertia - the spin–down which is one important process for the core flux expulsion, we will consider in this paper the field evolution for NSs modelled with three different EOS, covering the whole range from very stiff to soft ones.

However, the assumption that the field strengths at the NS's surface and in its core are initially of the same order of magnitude is not the only conceivable one. There are good reasons to assume that the field strengths at these positions differ considerably. This could be, e.g., a consequence of a field amplification during the collapse by flux conservation which results in a flux permeating the whole star with, say, 10^9 G and a very efficient field generation after the formation of the NS by the thermoelectric instability in the crustal layers, producing there, say, 10^{12} G. Thus, we want to investigate here the effect of different initial ratios $B_{p0}/B_{c0} \gg 1$, that is, initial magnetic configurations, for which the crust contains the bulk of the magnetic flux and only a tiny fraction of it is anchored in the core.

Another process which determines the initial MF structure in the new–born NS is the post core–collaps accretion of fall–back matter after the supernova explosion. If this accretion is hypercritical the ram pressure overwhelms the pressure of a possibly existent MF and submerges it. The depth of submergence depends on the total amount of accreted matter and on the EOS of the NS matter (for details see Geppert, Page & Zannias 1999). Given the parameters as estimated for SN 1987A (Chevalier 1989), a MF generated by dynamo action in the proto–NS or amplified simply by flux conservation during the collaps would be submerged down to the crust–core boundary or even into the core. The rediffusion of that submerged MF would last at least 10^8 years, thus, the NS born in SN 1987A will not appear as a pulsar in the near future, unless field generation processes in the crust will transform thermal into magnetic energy relatively fast. However, also fall–back much weaker than observed in SN 1987A will cause a certain submergence and, hence, a delayed switching on of pulsars (Muslimov & Page 1995). The discrepancy between the real age ($1.7 \cdot 10^5$ years) and active age ($1.6 \cdot 10^4$ years) of the pulsar B1757-24 (Gaensler & Frail 2000) can be explained at least partly by the assumption that the MF of this pulsar was submerged and rediffused during $\sim 10^5$ years (see Sec. 3). Generally, if there was initially some field generation in the whole NS, the post–supernova accretion onto the new born NS will lead to a field structure with $B_{p0}/B_{c0} \ll 1$, i.e. to a field structure having a much weaker MF strength at the surface than in the core. It is our aim to consider here the effect of different submergence depths onto the flux expulsion from the core and its consequences for the early NS evolution.

The paper is organized as follows. In Section 2 we give a short description of the model. It will be based on the model of the interplay of forces that determine the flux expulsion from the NS's core presented in detail in KG00. The numerical results for the different initial field structures are presented in Section 3 and Section 4 is devoted to the discussion and conclusions.

2   DESCRIPTION OF THE MODEL

We calculate the velocity of the flux carrying proton tubes (fluxoids) using the model described in detail in KG00. Namely, fluxoids are supposed not to be rigidly tied with the neutron vortices (Srinivasan et al. 1990, Jahan-Miri & Bhattacharya 1994), but to move under the common action of the buoyancy force, the drag force and the force exerted by the neutron vortices.

The superfluid core participates in the rotation of the NS by forming an array of quantized neutron vortices. The radial velocity of vortices at the crust-core boundary is determined by

$$v_n = -\frac{R_c \dot{\Omega}_s}{2\Omega_s}, \tag{1}$$

where Ω_s is the averaged angular velocity of the core superfluid, R_c is the radius of the NS core. In this paper we consider the evolution of isolated NSs. We assume, that the rotational evolution is determined by energy losses due to magneto–dipole radiation:

$$P\dot{P} = \frac{2B_p(t)^2 R^6}{3Ic^3}, \tag{2}$$

where B_p is the surface field strength at the magnetic pole, R is the radius of the NS, I its moment of inertia, c is the speed of light and the magnetic axis is perpendicular to the rotational one.

The buoyancy force, acting on the unit length of a fluxoid, is given by (Muslimov, Tsygan 1985):

$$f_b = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \frac{1}{R_c} \ln\left(\frac{\lambda}{\xi}\right), \tag{3}$$

where $\Phi_0 = 2 \times 10^{-7}$ G cm^2 is the quantum of the magnetic flux, λ is the London penetration depth, and ξ is the proton coherence length, which is less than $\lambda/\sqrt{2}$ for a superconductor of type II. The buoyancy force is always positive, i.e. directed outward.

The drag force per unit length of a fluxoid, resulting from the interaction of the normal electrons in the core with the magnetic field of a fluxoid (Harvey, Ruderman, Shaham 1986, Jones 1987), is given by

$$f_v = -\frac{3\pi}{64} \frac{n_e e^2 \Phi_0^2}{E_F \lambda} \frac{v_p}{c}, \tag{4}$$

where n_e is the number density of electrons in the core, which is assumed to be about 5% of the number density of neutrons, e is the elementary charge, E_F the Fermi energy of the electrons, and v_p is the velocity of fluxoid. For the Fermi energy and electron number density we take the values determined by the density at the crust–core interface, $\rho_c = 2 \cdot 10^{14}$g/cm^3. When $v_p > 0$ (fluxoids are moving outward), one has $f_v < 0$, i.e. f_v hampers the expulsion of the flux from the core of NS.

The force, acting on the unit length of a fluxoid exerted by the neutron vortices, is given by (DCC)

$$f_n = \frac{2\Phi_0 \rho r \Omega_s(t)\omega(t)}{B_c(t)}, \tag{5}$$

where $\omega = \Omega_s - \Omega_c$ is the lag between the rotational velocity of the core superfluid, Ω_s, and the rotational velocity of both the crust and the charged component of the core, Ω_c, while B_c denotes the averaged core magnetic field. The observed spin period P of a NS is related to Ω_c by $P = 2\pi/\Omega_c$. The maximum lag ω_{cr}, which can be sustained by the pinning force, defines also the maximum force, which can be exerted by the vortices on to the fluxoids (see DCC and Jahan–Miri 2000 for a more detailed discussion). According to DCC,

$$\omega_{cr} = 8.7 \times 10^{-2} x_p \alpha_g r_6^{-1} \left(\frac{m_p - m_p^*}{m_p}\right) \left(\frac{m_p^*}{m_p}\right)^{-1/2} \times$$
$$\times B_{c12}^{1/2} \ln\left(\frac{\lambda}{\xi}\right) \sin(2\chi) \text{ rad s}^{-1}, \tag{6}$$

where x_p is the fractional concentration of protons, α_g is a numerical factor of order of the unity, r_6 is the distance from the NS rotational axis in 10^6 cm, m_p is the mass of the proton, m_p^* its effective mass, B_{c12} is the core magnetic field strength in units of 10^{12} G, and χ is the angle between rotational and magnetic axis. We assume hereafter $x_p = 0.025$, $\alpha_g = 1$ and $m_p^* = 0.8 m_p$

Vortices can either move outward faster than fluxoids (forward creeping, $\omega = \omega_{cr}$), or the velocities of both kinds of flux tubes can coincide (comoving, $-\omega_{cr} < \omega < \omega_{cr}$), or neutron vortices can move slower than fluxoids (reverse creeping, $\omega = -\omega_{cr}$). The vortex acting force can be both positive or negative, depending on the sign of ω, i.e. f_n can either promote the expulsion or impede it.

The sum of the powers of the forces, acting onto the fluxoids in the core, is equal to the Poynting flux through the surface of the NS core:

$$\sum_{fluxoids} \int (f_b + f_n + f_v) v_p \mathrm{d}l = -\frac{c}{4\pi} \int_{S_{core}} \left[\vec{E} \times \vec{B}\right] \cdot d\vec{S}_{core}, \tag{7}$$

where the summation runs over all fluxoids. The integral on the l.h.s. of (7) is taken over the length of each fluxoid, while the integral on the r.h.s. of (7) is performed over the surface of the core; the normal vector of this surface is directed into the core. We consider the evolution of a poloidal field, which is supposed to be dipolar outside the NS. Introducing a vector potential $A = (0, 0, A_\phi)$ and the Stokes stream function $A_\phi = S(r,t)\sin(\theta)/r = B_{p0}R^2 s(r,t)\sin(\theta)/r$, where r, θ, ϕ are spherical coordinates, $s(r,t)$ the normalized stream function, and B_{p0} the initial magnetic field strength at the magnetic pole, one can express the field components as

$$B_r = B_{p0}R^2 \frac{s}{r^2}\cos\theta, \quad B_\theta = -\frac{B_{p0}R^2}{2}\frac{\sin\theta}{r}\cdot\frac{\partial s}{\partial r}. \tag{8}$$

By means of the variable s the integral on the r.h.s. of the equation (7) can be rewritten as:

$$\frac{c}{4\pi} \int_{S_{core}} \left[\vec{E} \times \vec{B}\right] \cdot d\vec{S}_{core} = \frac{B_{p0}^2 R_c^2}{6}\frac{\partial s(R_c,t)}{\partial t}\frac{\partial s(R_c,t)}{\partial r}. \tag{9}$$

Assuming the MF in the core to be uniform, one can introduce the total core forces

$$F_{n,b,v} = f_{n,b,v} \cdot 4R_c/3 \cdot N_p, \tag{10}$$

where $4R_c/3$ is the mean length of the fluxoid and $N_p = \pi R_c^2 B_c / \Phi_0$ is the number of fluxoids. It is worth mentioning that $F_n \propto B_c^{1/2}$ both in the forward and in the reverse creeping regimes, but $F_b, F_v \propto B_c$.

One can now introduce the crustal force

$$F_{crust} = \frac{B_{p0}^2 R^4}{6}\frac{1}{v_p}\frac{\partial s(R_c,t)}{\partial t}\frac{\partial s(R_c,t)}{\partial r}. \tag{11}$$

When $\partial s(R_c,t)/\partial r$ is positive and $\partial s(R_c,t)/\partial t$ is negative the crustal force is negative, i.e. it prevents the roots of the fluxoids to move towards the magnetic equator. The crustal force is proportional to B_c^2.

Now one can rewrite equation (7) as:

$$F_n + F_b + F_v(v_p) + F_{crust}(v_p) = 0, \tag{12}$$

which determines the velocity of fluxoid v_p.

The equation of the balance of electromagnetic energy in the region outside NS core reads:

$$\int_{V_{crust}} \frac{j^2}{\sigma}\, dV + \frac{d}{dt}\int_V \frac{B^2}{8\pi}\, dV + \frac{c}{4\pi}\int_{S_{core}} \left[\vec{E} \times \vec{B}\right] \cdot d\vec{S}_{core} = 0. \tag{13}$$

The first integral is restricted to the crust, the second has to be taken over all space excluding NS's core, and the third is a surface integral over the core. Therefore, the crustal force determined by the equation (11) coincides with the crustal force determined by the equation (8) from KG00, however, a smoother numerical procedure can be achieved with the recent version.

The evolution of the magnetic field in the solid crust is governed by the induction equation without a convective term:

$$\frac{\partial \vec{B}}{\partial t} = -\frac{c^2}{4\pi}\nabla \times \left(\frac{1}{\sigma}\nabla \times \vec{B}\right). \tag{14}$$

It can be rewritten in terms of the Stokes stream function as:

$$\frac{\partial s}{\partial t} = \frac{c^2}{4\pi\sigma}\left(\frac{\partial^2 s}{\partial r^2} - \frac{2s}{r^2}\right), \tag{15}$$

and should be supplemented by the boundary conditions at the surface ($r = R$) and at the crust-core boundary ($r = R_c$). While the outer boundary condition is given by $R\frac{\partial s}{\partial r} = -s$ at $r = R$, the inner one is time-dependent and determined by equation (12).

The conductivity in the regions of interest in the crust is determined by the scattering of electrons at impurities and phonons. The phonon conductivity is dependent on the temperature, and, hence, on the cooling history of the NS. The impurity conductivity is temperature independent and is determined by the concentration of the impurities Q. The relaxation times for both electron scattering processes depend strongly on the density and on the chemical composition of the crust. For the phonon conductivity we use the numerical data given by Itoh et al. 1993, for the impurity conductivity we apply the analytical expression derived by Yakovlev & Urpin 1980.

The evolution of the magnetic field in the superconducting core is driven only by the motions of the fluxoids:

$$\frac{\partial \vec{B}_c}{\partial t} = \nabla \times \left(\vec{v}_p \times \vec{B}_c\right). \tag{16}$$
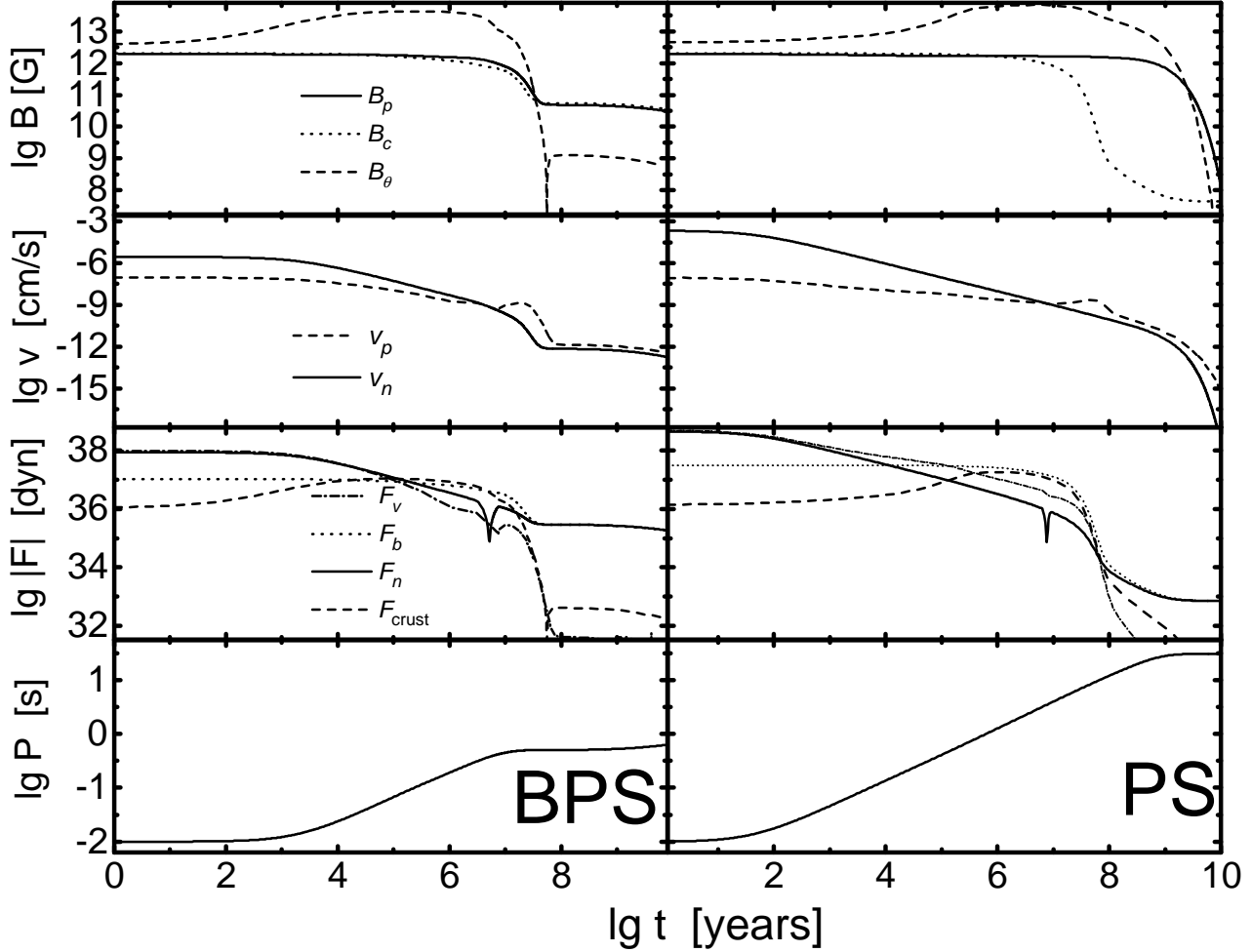
Figure 1. The evolution of the magnetic field strengths (surface field at the magnetic pole B_p, core field B_c, and θ-component of the field at the magnetic equator at $R = R_c$), velocities of vortices and fluxoids, forces and rotational period of a NS, based on BPS (left column) and PS (right column) EOS. Parameters: $Q = 0.1$, $P_0 = 0.01$, $B_{c0} = B_{p0} = 2 \cdot 10^{12}$ G.

This equation can be rewritten in terms of the normalized stream function s by:

$$\frac{\partial s}{\partial t} = -v_p \frac{\partial s}{\partial r}. \tag{17}$$

The assumption of the homogeneity of the magnetic field in the core leads to the following ansatz for the fluxoid velocity

$$v_p(r, t) = \alpha(t)r, \tag{18}$$

and for $s(r, t)$ in the core

$$s(r, t) = \frac{B_c(t)r^2}{B_{p0}R^2}. \tag{19}$$

With this, the solution of equation (17) describes the evolution of the core MF at the crust–core boundary as

$$B_c(t) = B_{c0} \exp\left(-2 \int_0^t \alpha(t')dt'\right). \tag{20}$$

Note, that in equations (15) and (17) of KG00 there are two misprints. However, the numerical computation was based on the correct equations. We use the procedure described in KG00 to find the velocity of the fluxoids and, consequently, the magnetic and rotational evolution of NS.

3   NUMERICAL RESULTS

Although the induction equation is linear, the problem, as shown in Section 2, is a nonlinear one in terms of the magnetic field, and has a time and field dependent boundary condition at the crust–core interface. Additionally, equation (12) which determines the flux expulsion velocity is nonlinear in v_p too. For the calculations described below we use the standard cooling scenario considered by Van Riper (1991) for the neutron star model based on a stiff (Pandharipande & Smith 1975), medium (Friedman–Pandharipande 1981) and a soft (Baym, Pethick & Sutherland 1971) EOS. The cooling histories for those EOS are taken from Van Riper (1991).

The main influence of the choice of the EOS on to the MF evolution comes via the different density profiles which determine the scale lengths of the MF in the corresponding NS models.

In all our model calculations we apply the initial rotational

period $P_0 = 0.01$ s, a value which is generally accepted for new–born NSs.

3.1 The effect of different EOS

In order to extract the effects of different EOS on to the flux expulsion from the NS's core we apply for all models the initial surface and core magnetic field strengths $B_{p0} = B_{c0} = 2 \cdot 10^{12}$ G, an impurity parameter $Q = 0.1$ and the standard cooling scenario.

We consider three qualitatively different EOS and use the corresponding density profiles calculated for a $1.4 M_\odot$ NS (for details see Urpin & Konenkov (1997) and references therein). The soft EOS is represented by the BPS model with a radius of 7.35km and a crust thickness of 310m. The corresponding values for the medium (FP model) and stiff (PS model) EOS are 10.61km, 940m, and 15.98km, 4200m, respectively. The effect of increasing stiffness of the EOS describing the state of the core matter is at least threefold: i) since $P\dot{P} \propto B_p^2 R^4/M$ the increasing radius of the NS R leads to a more efficient spin–down for a given surface MF B_p and NS mass M; ii) the larger scale of the MF and iii) the faster cooling of the NS cause a deceleration of the crustal field decay.

In Figure 1 we show the evolution (from top to bottom) of the MF, velocities of vortices and fluxoids, forces and rotational periods of NS models based on a very soft (BPS) and a very stiff (PS) EOS. As in the case of the medium FP EOS (see KG00), at the beginning the dominating expulsive force is F_n. This is because in the core of the magnetized and rapidly spinning NS there exists a large number of fastly outward moving neutron vortices, which act on the fluxoids. As shown in the third panel of Figure 1, during that relatively short early period of $t < 10^4 - 10^5$ years, F_n is balanced by F_v, while the other forces in eq.(12) are negligible. However, because of the preceding NS spindown, the number of vortices in the core (and, hence, F_n) decreases, and F_b becomes the main force, which drives the fluxoids outward. In KG00 we have shown, that in case of an intermediate EOS, the timescale of the expulsion of the magnetic field from the core τ_e is determined by the balance of F_b and F_{crust}. One can see that, although F_b and F_{crust} are still the dominant forces governing the expulsion timescale both for NSs with stiff and soft EOS, in the case of the BPS model one can not neglect F_n in the balance of forces. NSs with softer EOS spin down slower than NSs with stiffer EOS. In 10^6 years a NS with the soft EOS spins down to only 0.19 s, while a NS with the stiff EOS spins down up to 1.26 s. Thus, F_n, which is proportional to the number of vortices, i.e. inversely proportional to the spin period of the NS, plays a more important role in the dynamics of fluxoids for NSs with a soft EOS. The evolution of their surface MF follows closely the core field decrease, because the diffusion timescale of the crustal currents is smaller than the expulsion timescale of the core field. The expulsion of the flux in case of the BPS model stops at about $3.5 \cdot 10^7$ years, leaving behind a residual field $B_{res} \approx 5 \cdot 10^{10}$ G, since then the vortex acting force balances F_b and prevents further expulsion. Therefore, the slight spindown results in a relatively short residual spin period $P_f \approx 0.5$ s and a high strength of the residual MF.

In the case of a NS model with a stiff EOS, the expulsion timescale appears to be much shorter than the dissipation
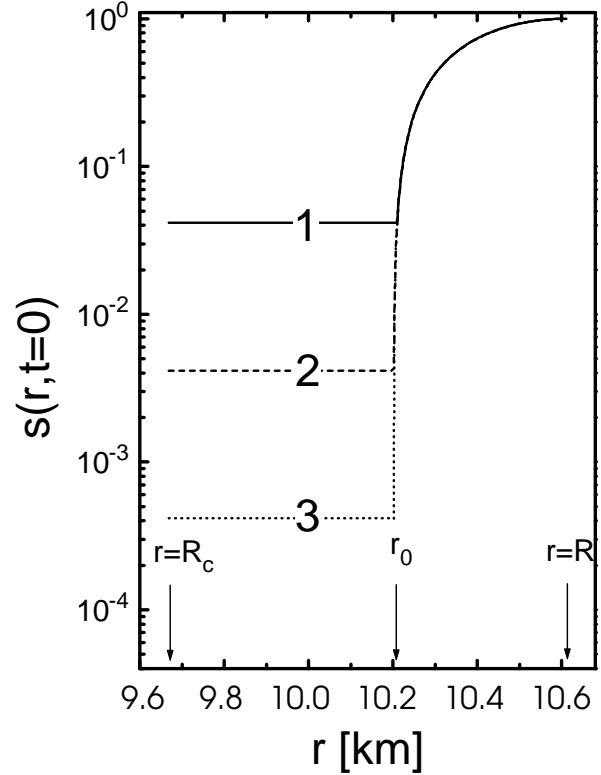

Figure 2. The initial profiles of the normalized stream function $s(r, t = 0)$ in the crust ($R_c < r < R$). Lines marked by 1, 2, 3 correspond to $B_{c0} = 10^{11}, 10^{10}$ and 10^9 G, respectively. The $r = r_0$ corresponds to the density $\rho = \rho_0$.

timescale of the field in the thick crust. When the MF in the core has been reduced by ~ 4 orders of magnitude (after $\sim 2 \cdot 10^8$ years), the surface field is still $\sim 10^{12}$ G. Thus, the evolution of the surface MF in NSs with stiff EOS is insensitive to the details of the physics of the superfluid core (but, of course, is sensitive indirectly via the cooling history of NS which is extremely sensitive to the occurrence of superfluidity). In order to check the sensitivity of the results with respect to another *a priori* unknown parameter of our models, we varied Q in the range $0.01 - 1$ and found, that the surface MF begins to decay for any Q when the core flux is almost completely expelled.

Since the NS with stiffening of the EOS spins down more effectively (because of the larger radius and the longer decay–timescale of the surface MF), at the end of the evolution a smaller number of neutron vortices remains in the core which can hold there only a smaller number of fluxoids than in case of a softer EOS NS model. Thus, the residual field becomes weaker with stiffening of the EOS.

3.2 The case of $B_{c0} << B_{p0}$

There exist some observational and theoretical evidences that the initial core field might be weaker than the surface one. Thus, Chau, Cheng & Ding (1992) concluded from an analysis of glitch observations in the Vela pulsar that the MF at the crust–core boundary is in the order of $10^8 - 10^9$ G while the surface MF is about three to four orders of
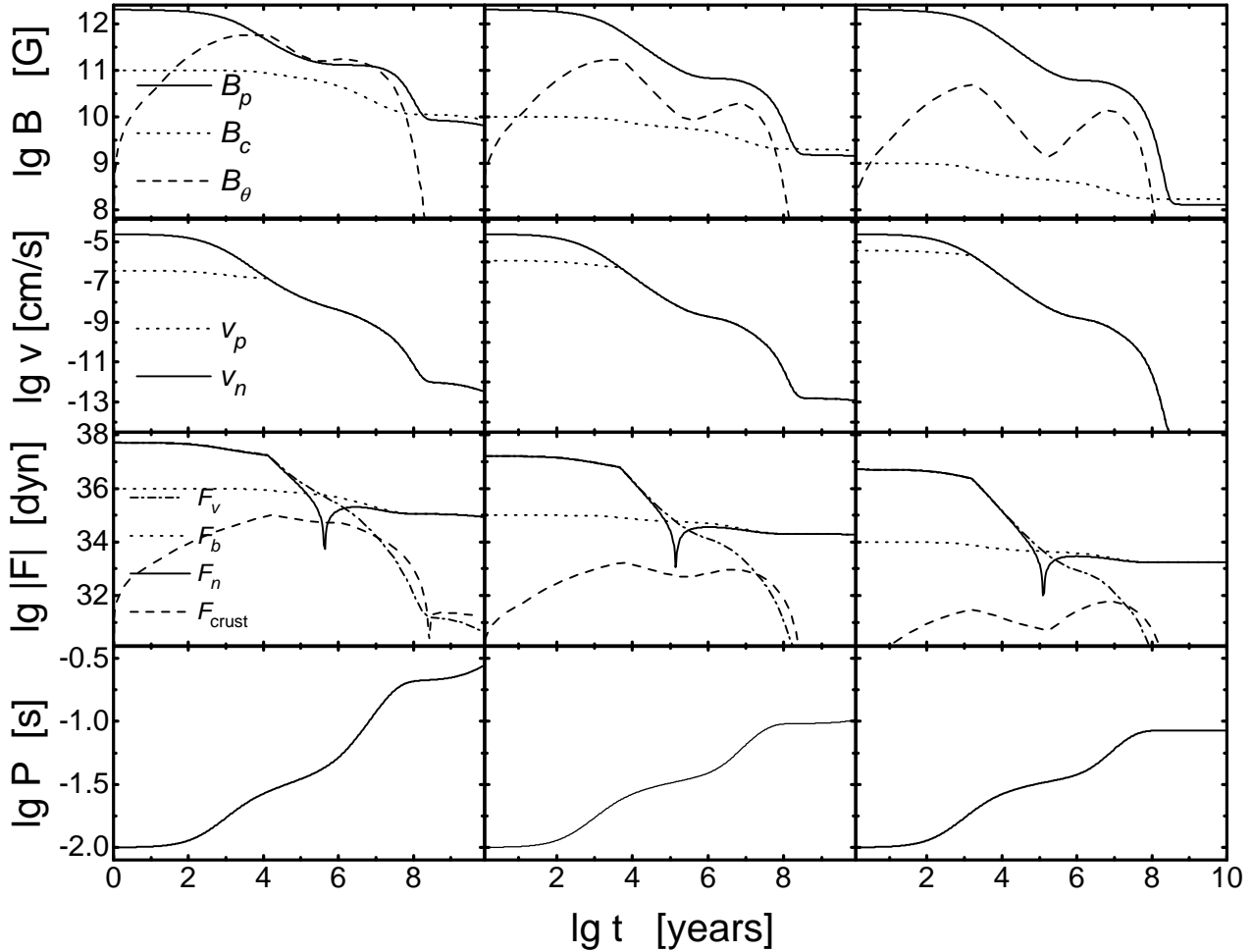
Figure 3. The evolution of the magnetic field strengths (surface field at the magnetic pole B_p, core field B_c, and θ-component of the field at the magnetic equator at the $R = R_c$), velocities of vortices and fluxoids, forces and rotational period of a NS, based on the FP EOS, for $B_{p0} = 2 \cdot 10^{12}$ G, $B_{c0} = 10^{11}$ G (left column), 10^{10} G (middle column) and 10^9 G (right column), for the initial s–profiles from Figure 2.

magnitude larger. Such a difference in the field strengths at the surface and at the crust–core boundary could be produced by the thermoelectric instability (Blandford, Applegate & Hernquist 1983, Urpin, Levshakov & Yakovlev 1986, Wiebicke & Geppert 1996), which may transform thermal into magnetic energy effectively during the very early period of NS's life in its outermost crustal layers.

In Figure 2 we show the initial field configurations in terms of the Stoke's stream function $s(r, t = 0)$ for $B_{p0} = 2 \cdot 10^{12}$ G and $B_{c0} = 10^9, 10^{10}$, and 10^{11} G. The calculations have been performed for an initial period $P_0 = 0.01$ s, an impurity parameter $Q = 0.1$ and an initial density where the crustal field matches the inner one, $\rho_0 = 10^{13}$g cm^{-3}. In order to show clearly the effect of different B_{p0}/B_{c0}, we will not vary Q and ρ_0 as well as present the results for the medium FP EOS and standard cooling only.

We show in Figure 3 the evolution of the field components, of the forces acting on to the fluxoids in the core, of the velocities of both the fluxoids v_p and of the neutron vortices v_n, and the rotational evolution in terms of the rotational period $P(t)$.

Since $F_{crust} \propto B_c^2$, $F_b, F_v \propto B_c$, but $F_n \propto B_c^{1/2}$, a relative decrease of the field strength in the core reduces F_{crust}, F_b and F_v much faster than F_n, thereby enforcing the role the vortices play for the dynamics of the fluxoids for the weaker B_c. As seen from the third panels of Figure 3 apart from the early stages of evolution ($t = 10^3...10^4$ years) the fluxoids are completely in the comoving regime, and their motion is totally determined by those of the neutron vortices, and the velocity of fluxoids at $t = 0$ is the greater, the smaller B_{c0} is. Consequently, a MF configuration with $B_{c0} << B_{p0}$ results in a "spin–down induced" flux expulsion as described by Jahan-Miri & Bhattacharya (1994) for accreting NSs, and the influence of the crustal force on the dynamics of fluxoids is negligible.

The decay of the surface MF almost fully coincides with that in case of the purely crustal magnetic configurations (Urpin & Konenkov 1997, Page et al. 2000). Only in the case of $B_{c0} = 10^{11}$ G, the value of B_p during the plateau phase at $t = 10^5 - 10^7$ years is slightly higher than in the case of $B_{c0} = 10^{10}$ or even 10^9 G. However, the most remarkable difference is that the surface MF decays not down to zero,
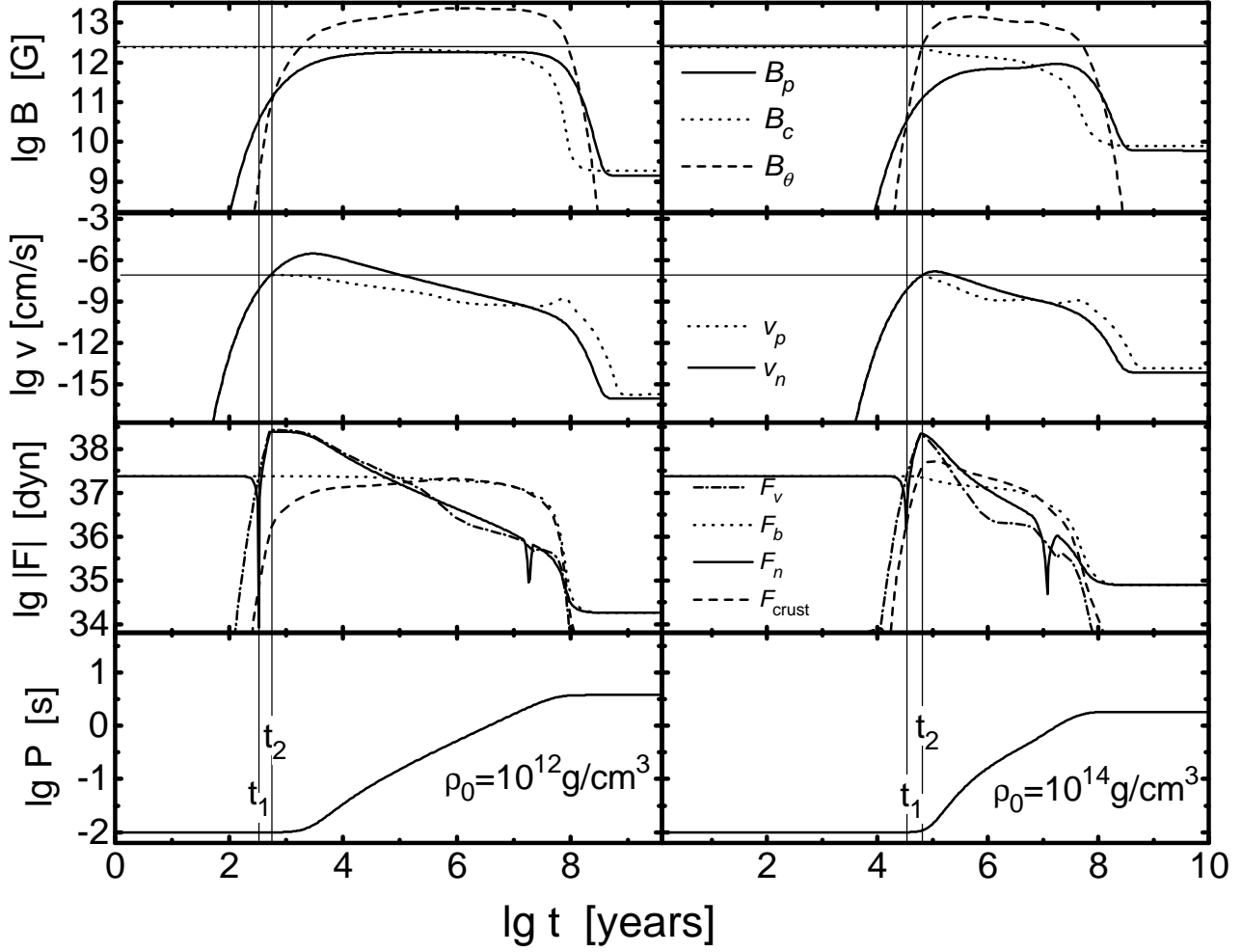
Figure 4. The same as in Fig. 1 for the initially submerged field.

but to a nonzero residual value. Since $v_p = v_n$ after $t > 10^4$ years the ceasing of the spin–down of the NS, when B_p has been decayed down to the weak B_c, results in a ceasing of the further field expulsion from the core. Thus, in the case of $B_{c0} = 10^{11}, 10^{10}$ and 10^9 G, $B_{res} \approx 10^{10}, 2 \cdot 10^9$ and $1.5 \cdot 10^8$ G, respectively.

It is worth mentioning that the choice of a stiffer EOS leads to a more extended phase of forward creeping. For such stiff EOS and relatively strong core fields $(B_{c0}/B_{p0} \geq 0.1)$ after about 10^8 years even the reverse creeping stage can be entered.

3.3 The effect of different submergence depths

A possible explanation for the discrepancy between the estimated rates of supernovae and of pulsar births in the galaxy (Frail 1998) is the delayed switch–on of a pulsar. This feature has been discussed by Muslimov & Page (1995), who considered a relatively shallow submergence of the initially present MF. However, the immediate consequence of a supernova (type Ib or II) explosion will be a fall back of matter on to the new born NS (Colpi, Shapiro & Wasserman, 1996). In case this fall back accretion is hypercritical the ram pres-

sure of the back–falling matter is larger than the pressure of a, say, 10^{12} G initially present MF and the accretion flow is purely hydrodynamic; conditions as could be estimated for the SN 1987A (see Chevalier 1989). Applying the time dependence of the fall back accretion rates given by Colpi et al. (1996), Geppert, Page & Zannias (1999) calculated the submergence of a NS MF for different EOS as a function of the total amount of accreted matter ΔM. It became clear that for values as inferable for SN 1987A (Chevalier 1989) the MF has been submerged so deeply that the created NS will not shine up as a radiopulsar for more than 10^8 years. When the fall back accretion ceases the submerged field will start the rediffusion towards the surface of the NS. Once the NS is an isolated one during all its life, for given conductive properties of the layers above the field maintaining electrical currents and a given EOS determining the cooling and the scalelength of the field, the submergence density ρ_{sub} down to which the MF has been buried by fall back accretion is the only parameter which defines the duration of that rediffusion process τ_{red}. Evidently, the larger ΔM the larger ρ_{sub} and, hence, the larger τ_{red}.

In order to show clearly the effect of the different ρ_{sub} we will use the same representation as in the preceding section for the medium EOS with $Q = 0.1$. In that way Figure 4

shows the evolution of the NS for $\rho_{sub} = 10^{12}$, and 10^{14}g cm^{-3}, which corresponds roughly to an total amount of accreted matter of about 3×10^{-5}, and $10^{-2}\ M_\odot$, respectively. The process of flux expulsion from a NS with initially submerged MF is qualitatively different from that of a standard NS. Since at the beginning of the NS's life, after the fall back has finished, the surface MF B_{p0} is practically zero, there is no spin–down and, hence, the velocity of the vortices v_n is zero too. Thus, the neutron vortices, because being fixed in number and location, will not force the fluxoids to move outward, but will impede them to be expelled by the buoyancy force. The immobility of the fluxoids and the initial smallness of the crustal forces results in a situation where the balance of forces is given only by $F_b + F_n = 0$, where the vortex acting force F_n is just compensating the buoyancy, while F_v and F_{crust} are many orders of magnitude smaller than F_n, F_b.

When the field would remain buried there were no chance to expel the core flux because the number of vortices remains constant in a stationary rotating NS. However, depending on ρ_{sub} the submerged crustal field rediffuses towards the surface thereby causing an increasing braking of NS's rotation. As shown in Figure 4 that the spin–down onsets the later the deeper the crustal field has been submerged. The initially immobile neutron vortices start to move outward and the proton fluxoids, tightly bounded to them by F_b follow their motion towards the crust, i.e. the fluxoids are in the comoving state. With increasing v_p the drag force F_v rises and, since the crustal force is still negligible, the balance of forces is determined by $F_b + F_n + F_v = 0$. During that early epoch the core field does not change which results in a constant and positive F_b. Since, on the other hand both F_v and F_n are negative, the increasing absolute value of drag force reduces the absolute value of the vortex acting force. By the vertical line with mark t_1 we indicated the moment, when $F_n = 0$; t_1 is $3 \cdot 10^2$ years for $\rho_{sub} = 10^{12}$g cm^{-3} and $6 \cdot 10^4$ years for $\rho_{sub} = 10^{14}$gcm^{-3}. Exactly at that moment there is no interaction between the fluxoids and the vortices and the balance of forces is given by $F_b + F_v = 0$.

However, the rediffusion of the crustal field continues beyond $t = t_1$ and the velocity v_n (and hence v_p) increases too. The change in the sign of F_n reflects the change in the state of flux expulsion: while early on the vortices hampered the movement of the fluxoids, now the positive F_n supports the flux expulsion, though it is still $v_n = v_p$. The process of increasing F_v and F_n proceeds until F_n reaches its maximal value, determined by $F_n = F_n(\omega_{cr})$. This moment is indicated in Figure 4 by t_2, which is $6 \cdot 10^2$ years for $\rho_{sub} = 10^{12}$g cm^{-3} and $7 \cdot 10^4$ years for $\rho_{sub} = 10^{14}$gcm^{-3}. At that point the fluxoids can no longer follow the motion of the vortices, which cut now through the fluxoids, hence, the forward creeping regime has been reached. Close to this moment, the crustal force becomes comparable to F_b, however, both are small in comparison with F_n and F_v. Later on, when the surface MF B_p becomes comparable to the core field B_c, the transition from a "submerged" to a "standard" field configuration has been performed. The velocity of fluxoids is no longer determined by F_n and F_v, but instead by F_{crust} and F_b , a situation which is the same as described in KG00 and the evolution goes through the phases of the forward creeping, the comoving, and the reverse creeping regime. It turns out, that the long–term magnetic and rotational evolution does not depend crucially on ρ_{sub}; it affects mainly the NS behaviour in its early life and may lead to the phenomenon that a young NS is recognized to be an old one because its active age, $\tau_a = P/2/\dot{P}$, is apparently high. However, especially for very deep submergence of the field, the pulsar may also turn to look younger than its real age when the rediffusion progresses (see Geppert et al 1999). The rediffusion of the initially submerged MF may lead to some interesting metamorphoses of ages of NSs: the young NSs look older or younger than they really are. For example, from the magnetic evolution of the NS at the right panel of Figure 4, one can estimate at $t = 130000$ years an active age of about 50000 years, and a braking index $n \equiv 2 - \ddot{P}P/\dot{P}^2 \approx 1.4$. The pulsar B1757-24 in SNR G5.4-1.2 has an active age of 16000 years and a braking index $n < 1.33$, while its real age was estimated from the morphology of the SNR to be 170000 years (Gaensler & Frail 2000). Obviously, the assumption of MF submergence in NSs can at least partly explain the discrepancy between true and active age, and the occurence of a small braking index for this pulsar (Muslimov & Page 1995).

The other process which generates a strong surface field after the NS's birth, the thermoelectric instability, may not account for those apparent discrepancies in the ages, because this instability acts effectively at most during the first few thousand years of the NS's life.

Note however, that with increasing ρ_{sub}, B_{res} increases while P_{res} decreases. An increase of ρ_{sub} from 10^{12}g cm^{-3} to 10^{14}gcm^{-3} causes an increase of B_{res} by a factor of 4 and decreases P_{res} by a factor of 2. This can be understood from the fact that for larger ρ_{sub} the spin–down starts later. During that longer period a small amount of core flux has been expelled and dissipated in the inner crust as well as the NS cooled down, thereby enhancing the conductivity in the surface layers which decelerates the rediffusion. Thus, in the process of rediffusion the surface MF can not reach the same high value as for the corresponding case with shallow submergence. Therefore, the spin–down in case of the deep submerged MF is less efficient, P_{res} is smaller, which reduces the efficiency of the flux expulsion from the core, resulting eventually in a larger B_{res}.

3.4 Shear stresses induced by flux expulsion

In this paper the crust of the NS is assumed to be solid and not deformed by Maxwell (and others) stresses, i.e. the crustal MF evolves only through Ohmic dissipation and does not affect the elastic properties of the crust. The opposite case was considered by Ruderman (1991a,b) and Ruderman, Zhu and Chen (1998), who investigated the consequences of the changing core MF configuration for the spin history, especially glitches. Moving the roots of the fluxoids at the crust–core interface, crustal strains are built up and relaxed by large–scale crust–cracking events. From our calculations we can estimate the shear stress at the crust–core boundary. The total force, acting upon the crust by the expelled flux is balanced by the force, acting from the crust on the fluxoids, i.e. F_{crust}. The maximum of $|F_{crust}|$ is reached at that stage of evolution, when the balance of F_b and F_{crust} determines the velocity of fluxoids. It seems to be a rather common situation, that the maximum force, which acts on the crust,
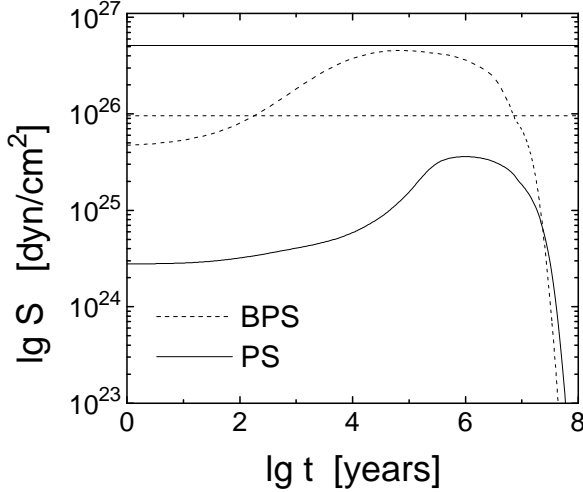
Figure 5. Temporal evolution of the shear stresses at the bottom of the crust generated by the movement of the fluxoids, calculated by use of equation (21) for the soft and the stiff EOS. The maximum stresses, given by equation (22) for $\mu = 2 \cdot 10^{29}$ dyn/cm^2 and $\theta_{max} = 10^{-2}$, are shown as horizontal lines.

does not exceed greatly the buoyancy force, which acts on the fluxoids from the beginning of the NS evolution. This force is about 10^{37} dyn, as one can see from Figure 1, for $B_{c0} = 2 \cdot 10^{12}$ G. The shear stress exerted at the base of the crust is given by

$$S_{crust}(t) \sim \frac{|F_{crust}(t)|}{R_c \Delta R}, \tag{21}$$

where ΔR is the thickness of the crust and $|F_{crust}(t)|$ is evaluated by use of equation (11). In Figure 5 we compare S_{crust}, calculated for $B_{c0} = 2 \cdot 10^{12}$G and $Q = 0.1$, with the maximum shear stress the crust can sustain before yielding, as estimated by (Ruderman 1991a),

$$S_{max} \sim \frac{\Delta R}{R} \mu \theta_{max} \lesssim 3 \cdot 10^{26} \text{dyn/cm}^2, \tag{22}$$

where θ_{max} is the elastic strain limit, $\theta_{max} \lesssim 10^{-2}$ for a $10^6 - 10^8$ years old NS, and $\mu \lesssim 10^{30}$ dyn cm^{-2} is the shear modulus of the crust. It is seen that the shear stresses reach their maximum after $10^4 - 10^6$ years and become negligible when the magnetic flux is expelled from the core. Clearly, the probability for crust–cracking increases with a softening of the EOS. Therefore, while for NSs with a soft EOS early crust–cracking events, as glitches, may occur, their appearance in young ($t < 10^5$ yrs) NSs with stiffer EOS hints to an initial field strength larger than $2 \cdot 10^{12}$ G and/or to a smaller yield strain ($\sim 10^{-4}$) of the young NS crust.

4 DISCUSSION AND CONCLUSION

In order to investigate the magnetic and rotational evolution of isolated NSs whose MF penetrates both its core and crust, we solved the system of equations which describe the expulsion of the MF from the superfluid core and its subsequent Ohmic dissipation in the solid crust. Thereby we assumed that the spin–down is driven by magneto–dipole radiation.

In a recent paper (KG00) we considered this process for a NS whose state of matter is described by a medium EOS and discussed the effects of different initial field strengths and impurity concentrations in the crust. For initial field strengths larger than 10^{12} G, our study yielded a decay time for the surface field $> 10^7$ years. In the present paper we considered the effect of different EOS on the MF evolution and its consequences. We studied also qualitatively different initial MF structures.

In the case of a configuration, where the initial MF strengths at the surface and in the core are of the same order of magnitude (modelled in this paper by $B_{p0} = B_{c0}$) we have found, that the main force, which drives the fluxoids outward, is the buoyancy force F_b. The expulsion timescale is determined by the balance of F_b and F_{crust}, which depend mainly on the core field strength and on the crustal conductivity, and is not affected by the spin history of the NS. The surprising result is that the expulsion timescale depends only weakly on the EOS. Since stiffer EOS results in a thicker crust, one would expect, that F_{crust}, which depends on the rate of dissipation of crustal currents, counteracts the expulsion of the magnetic flux from the core of a NS with the stiffer EOS for a longer time. However, the total core forces are proportional to R_c^3 (see eq. (10)), i.e., the total buoyancy force is about 6 times stronger for the PS-model than for the BPS model. The interplay of these effects results in rather similar flux expulsion timescales for different EOS.

However, the evolution of the observable surface MF differs substantially for soft and stiff EOS. For the soft BPS and medium FP (see KG00) EOS the diffusion timescale of the crustal field appears to be much shorter than the core flux expulsion timescale. Thus, the evolution of the surface and core MFs are tightly connected. Moreover, for the soft and the medium EOS the decay timescale of the surface MF depends on its initial strength. In the case of the thick crust as modelled by the NS with the PS EOS we have found, that the diffusion of the crustal field lasts longer than the expulsion of the core field by approximately two orders of magnitude. That means, in case of a stiff EOS the surface field evolution is not sensitive to the details of physics of the forces acting upon the fluxoids in the core, and does not depend strongly on the initial MF strength. Hence, for a stiff EOS the evolution of the surface MF coincides with that, considered by Konar & Bhattacharya (1999), for an initially expelled flux. This result is valid for initial surface MFs in the range of $10^{11} - 10^{13}$ G, and for impurity parameters in the range of $0.01 < Q < 1$.

It is generally accepted that the MF of normal radiopulsars decays weakly during their lifetime (Bhattacharya et al. 1992, Hartman et al. 1996). Unfortunately, the results obtained from our scenario of field evolution do not allow to select or to reject some EOS as being not in accordance with observational facts. All models lead to decay timescales of the surface field comparable or exceeding the radiopulsars lifetime.

As in the case of the medium EOS (see KG00), not the entire magnetic flux is expelled from the core, but some part remains there for eternity, resulting in a residual MF of NSs. This can be important for the explanation of the long-living MFs of millisecond pulsars. However, for those NSs a more complex analysis is required, which takes into account all the effects of accretion occuring in binary systems.

For a "crustal" MF configuration, when initially almost the whole magnetic flux is confined to the NS's crust, we have found that the neutron vortices play a very important role for the dynamics of the fluxoids. The velocity of the fluxoids appears to be equal to that of the vortices during almost the whole evolution, so that the flux expulsion can be really called "spin–down induced"(Srinivasan et al. 1990, Jahan Miri & Bhattacharya 1994). However, the evolution of the surface MF coincides almost completely with that of a purely crustal field as considered by Urpin & Konenkov (1997), and Page et al. (2000). Correspondingly, all conclusions of those papers are applicable to this magnetic configuration, namely the conclusion, that models of NSs, based on the medium and stiff EOS with standard cooling yield a satisfactory agreement with observations.

For a NS with an initially submerged crustal field we found that the submergence affects the evolution only during the early periods of the NS's life, as long as the crustal field is diffusing back to the surface. Then, the further magnetic and rotational evolution of the NS becomes similar to the "standard" evolution as described in KG00. The only reminder of the submergence episode is the relatively weak correlation of the residual field strength with the submergence depth and its anti–correlation with the final rotational period.

We also calculated the shear stresses at the bottom of the crust, which arise when the footpoints of the fluxoids are moved along the crust–core interface. We found that total force, which acts on the crust from the moved fluxoids (being equal to the force, which acts from the crust on to the fluxoids, i.e. F_{crust}), does not exceed strongly the buoyancy force, which acts onto the fluxoids from the very beginning of the evolution. It can be strong enough to break the crust, especially when a soft EOS decribes the state of the core matter and the crust is relatively thin.

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